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REPRESENTATIONS OF GL(n) OVER A p-ADIC FIELD WITH AN IWAHORI-FIXED VECTOR

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ABSTRACT
An elementary proof of Zelevinsky's classification for representations of GL(n) with an Iwahori-fixed vector is given using the theory of Hecke algebras.

Let G be the group of F-rational points of a connected, reductive, algebraic group over a p-adic field and let I be an Iwahori subgroup of G. The Hecke algebra $\mathcal{H}$ of compactly supported functions on G which are right and left invariant under I is a finitely-generated algebra which can be given explicitly in terms of generators and relations. It is also known that there is an equivalence between the category of admissible representations of G which are generated by their spaces of I-fixed vectors and the category of finite-dimensional $\mathcal{H}$-modules (this is a theorem of Bernstein, Borel, and Matsumoto). Therefore a special class of representations of G can be approached through the study of the explicity given algebra $\mathcal{H}$.

In this paper we give a proof of the classification theorems for irreducible $\mathcal{H}$-modules for the case $G = \mathrm{GL}_n(F)$ using the methods developed in [4]. The results proved here are special cases of results of A. Zelevinsky ([5]) on representations of $\mathrm{GL}_n(F)$. However, Zelevinsky's proofs make essential use of the group $\mathrm{GL}_n(F)$, whereas the methods used here refer only to the algebra $\mathcal{H}$. Thus, the parameter q which enters into the defining relations of $\mathcal{H}$ is constrained to be a power of a prime in Zelevinsky's work, while the proofs given here apply to more general values of q. Nevertheless, many ideas used here come from Zelevinsky's work.

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Since the completion of this paper, some very important work of Lusztig, Kazhdan and Lusztig, and Ginsburg has appeared in pre-print form. This work, motivated by conjectures of Lusztig, gives a K-theoretic approach to the construction of irreducible modules over Hecke algebras for general groups. The reader is also referred to recent pre-prints of Howe, Waldspurger–Moeglin, and Waldspurger, which deal with generalized Hecke algebras associated to super-cuspidal representations.

Throughout this paper, F denotes a p-adic field with ring of integers $\mathcal{O}$. Let π be a fixed prime element in F and let G denote $\mathrm{GL}_n(F)$. Let $q = \mathrm{Card}(\mathcal{O}/(\pi))$.

§1. The Hecke algebra for G

The symmetric group S_n will be denoted by W and the set of generating transpositions $\{s_1, \ldots, s_{n-1}\}$, with $s_j = (j, j+1)$, will be denoted by S. The pair (W, S) is a Coxeter group and its associated Hecke algebra is the $\mathbf{C}$-algebra with generators $\{T_w : w \in W\}$ and relations:

$$T_x T_y = T_{xy} \qquad \text{if } l(xy) = l(x) + l(y),$$

$$T_{s_j}^2 = (q-1)T_{s_j} + q, \qquad j = 1, \ldots, n-1$$

where $l : W \to \mathbf{Z}^+$ is the length function on W relative to S. This algebra will be denoted by $\mathcal{H}_W$.

Let $\mathcal{A} = \mathbf{C}[x_1, \ldots, x_n, x_1^{-1}, \ldots, x_n^{-1}]$ be the algebra of Laurent polynomials in $x_1, \ldots, x_n$. The group W acts on $\mathcal{A}$ by permuting the variables: $wx_j = x_{w(j)}$. Let $\mathcal{H}$ be the algebra generated by $\mathcal{H}_W$ and $\mathcal{A}$ subject to the relations:

$$x_i T_{s_j} = T_{s_j} x_i \qquad \text{if } |i-j| > 1,$$

$$x_i T_{s_i} = T_{s_i} x_{i+1} - (q-1)x_{i+1},$$

$$x_{i+1} T_{s_i} = T_{s_i} x_i + (q-1)x_{i+1}.$$

Every element of $\mathcal{H}$ has unique expressions of the form

$$T = \sum_{w \in W} a_w T_w = \sum_{w \in W} a'_w$$

for some a_w, $a'_w \in \mathcal{A}$.

By a theorem of Bernstein and Zelevinsky, $\mathcal{H}$ is isomorphic to the Hecke algebra of G with respect to an Iwahori subgroup. More precisely, let

$$I = \{(g_{ij}) \in \mathrm{GL}_n(\mathcal{O}) : g_{ij} \in \pi\mathcal{O} \text{ if } i > j\}$$

be the standard Iwahori subgroup of G and let $C_c(G//I)$ denote the algebra of compactly supported functions f and G such that $f(xgy) = f(g)$for all $g \in G$ and $x_1 y \in I$. The product of two such functions f_1 and f_2 is given by convolution:

$$f_1 * f_2(g) = \int_G f_1(h) f_2(h^{-1}g) dh$$

where dh is the Haar measure on G such that meas$(I) = 1$.

THEOREM 1.1 (Bernstein–Zelevinsky). *The algebra $\mathscr{H}$ is isomorphic to $C_c(G//I)$.*

A similar description, also due to Bernstein and Zelevinsky, of Hecke algebras of more general group is also valid. Proofs can be found in [2].

Let $\mathscr{C} = \mathrm{Hom}(\mathscr{A}, \mathbf{C})$. A character $\chi \in \mathscr{C}$ will be identified with a sequence $\chi = [\chi_1, \ldots, \chi_n]$ of non-zero complex numbers defined by: $\chi(x_i) = \chi_1$. The group W acts on $\mathscr{C}$ in the usual way: $w_\chi(x) = \chi(w^{-1}(x))$ for $w \in W$, $\chi \in \mathscr{C}$, and $x \in \mathscr{A}$.

To each partition $(n_1, \ldots, n_t)$ of n we associate a subset T of S as follows: $s_j \in T$ if $j \neq \sum_{k=1}^{l} n_k$ for all $l = 1, \ldots, t$. Let W_T denote the (parabolic) subgroup of W generated by T and let $\mathscr{H}_T$ denote the subalgebra of $\mathscr{H}$ generated by $\mathscr{A}$ and $\{T_w : w \in W_T\}$. Then $\mathscr{H}_T$ is clearly isomorphic to the Hecke algebra (with respect to an Iwahori subgroup) of $\mathrm{GL}_{n_1}(F) \times \cdots \times \mathrm{GL}_{n_t}(F)$. Denote the longest element in W_T by w_T.

§2. $\mathscr{H}$-Modules

Let $\mathscr{M}$ denote the category of finite-dimensional $\mathscr{H}$-modules. Throughout, all $\mathscr{H}$-modules will be assumed finite-dimensional.

For $M \in \mathscr{M}$ and $\psi \in \mathscr{C}$, set:

$$M_\psi = \{m \in M : xm = \psi(x)m \text{ for all } x \in \mathscr{A}\},$$

$$M_\psi^{\mathrm{gen}} = \{m \in M : (x - \psi(x))^t m = 0 \text{ for all } x \in \mathscr{A}, \text{ some } t \in \mathbf{Z}^+\},$$

$$P(M) = \{\psi \in \mathscr{C} : M_\psi \neq 0\}.$$

Elements of $P(M)$ will be called weights of M. We have:

$$M = \bigoplus_{\psi \in P(M)} M_\psi^{\mathrm{gen}}.$$

For each $\chi \in \mathscr{C}$, we define an $\mathscr{H}$-module $I(\chi)$ explicitly as follows As a basis for $I(\chi)$, we take elements ϕ_w for $w \in W$ and let $\mathscr{H}_W$ act on $I(\chi)$ by the left regular representation: $\phi_w = T_w \phi_1$, $T_w \phi_y = T_w T_y \phi_1$, for $y, w \in W$. The action of

$\mathscr{A}$ on $I(\chi)$ is uniquely determined by the condition: $x\phi_1 = \chi(x)\phi_1$ for all $x \in \mathscr{A}$. By the relations between $\mathscr{H}_W$ and $\mathscr{A}$ given in §1, for all $x \in \mathscr{A}$ and $w \in W$ there are unique elements $a_{y,w,x} \in \mathscr{A}$ such that

$$xT_w = T_w w^{-1}(x) + \sum_{y<w} T_y a_{y,w,x}$$

where $<$ denotes the Bruhat order on W with respect to S. In $I(\chi)$, therefore

$$x\phi_w = \chi(w^{-1}(x))\phi_w + \sum_{y<w} \chi(a_{y,w,x})\phi y.$$

It will be convenient to identify the underlying space of $I(\chi)$ with $\mathscr{H}_W$ (via $\phi_w \leftrightarrow T_w$); this should not cause any confusion.

For $\chi \in \mathscr{C}$, let $W_\chi = \{w_\chi : w \in W\}$ denote the W-orbit of χ. As noted in [4], $P(I(\chi)) = W\chi$ and an irreducible $\mathscr{H}$-module M is a quotient of $I(\chi)$ if and only if $\chi \in P(M)$.

We now recall some notation and results from [4]. For $w \in W$, let C_w and C'_w denote the Kazhdan–Lusztig elements of $\mathscr{H}_W$ associated to w. These elements of the Hecke algebra are defined in [1]. For $\chi = [\chi_1, \ldots, \chi_n] \in \mathscr{C}$, set

$$m_{s_i}(\chi) = q^{1/2}\left(\frac{\chi_i - q^{-1}\chi_{i+1}}{\chi_i - \chi_{i+1}}\right),$$

$$A_{s_i}(\chi) = m_{s_i}(\chi) + C_{s_i} = m_{s_i}(s_i\chi) + C'_{s_i}$$

($m_{s_i}(\chi)$ and $A_{s_i}(\chi)$ are defined only if $\chi_i \neq \chi_{i+1}$). For $w \in W$ with reduced decomposition $w = s_{i_1} \cdots s_{i_k}$ ($s_{i_j} \in S$), set

$$A_w(\chi) = A_{s_{i_1}}(s_{i_2} \cdots s_{i_k}\chi) \cdots A_{s_{i_{m-1}}}(s_{i_m}\chi) A_{s_{i_m}}(\chi).$$

Then $A_w(\chi)$ is an element of $\mathscr{H}_W$ whose coefficients with respect to a basis depend rationally on $\chi_1, \ldots, \chi_n$ and it does not depend on the choice of reduced decomposition of w. Whenever $A_w(\chi)$ is defined, $A_w(\chi)M_\chi \subseteq M_{w_\chi}$ for all $M \in \mathscr{M}$ and, viewed as an element of $I(\chi)$, $A_w(\chi) \in I(\chi)_{w_\chi}$. Furthermore, $A_w(\chi)$ exists and is invertible whenever $m_{s_{i_j}}(s_{i_{j+1}} \cdots s_{i_k}\chi) \neq 0, \infty$ for $j = 1, \ldots, k$.

Following Zelevinsky ([5]), a sequence of the form $\Delta = [q^{a-1}z, q^{a-2}z, \ldots, z]$ with $a \in \mathbf{Z}$ and $z \in \mathbf{C}^*$ is called a *segment*. Set $|\Delta| = a$ and let $\tilde{\Delta} = [z, qz, \ldots, q^{a-1}z]$.

Let $(n_1, \ldots, n_t)$ be a partition of n and let T be the subset of S associated to the partition as in §1. Let $\Phi = \{\Delta_1, \ldots, \Delta_t\}$ be a collection of segments such that $|\Delta_j| = n_j$ and let $\chi(\Phi) = (\Delta_1, \ldots, \Delta_t)$ and $\tilde{\chi}(\Phi) = (\tilde{\Delta}_1, \ldots, \tilde{\Delta}_t)$ denote the elements of $\mathscr{C}$ obtained by juxtaposing the Δ_j and $\tilde{\Delta}_j$, respectively. Let $I(\Phi)$ denote the

$\mathcal{H}_W$-submodule of $I(\tilde{\chi}(\Phi))$ generated over $\mathcal{H}_W$ by C_{w_T}. Here we are regarding C_{w_T} as an element in the underlying space of $I(\tilde{\chi}(\Phi))$. It is shown in §4 of [4] that $I(\Phi)$ is stable under all of $\mathcal{H}$ and that C_{w_T} is an element of weight $\chi(\Phi)$. Furthermore,

$$P(I(\Phi)) = \{w\chi(\Phi) : w \in W, l(ww_T) = l(w) + l(w_T)\}.$$

We call C_{w_T} the canonical generator of $I(\Phi)$. The $\mathcal{H}$-module $I(\Phi)$ corresponds to a representation of G obtained by inducing from a parabolic subgroup of type $(n_1, \dots, n_t)$, a product of special representations of the Levi factor $\mathrm{GL}_{n_1}(F) \times \cdots \times \mathrm{GL}_{n_t}(F)$. It will be convenient to also use $\Delta_1 \times \cdots \times \Delta_t$ to denote $I(\Phi)$. Furthermore, set:

$$W(T) = \{w \in W : l(ww_T) = l(w) + l(w_T)\}.$$

§3. The classification theorems

We state the theorems which give the classsification of irreducible $\mathcal{H}$-modules in this section. This will require some preliminary definitions.

DEFINITION 3.1. (i) For $z \in \mathbf{C}^*$, let L_z denote the set of sequences of the form $[\chi_1, \dots, \chi_m]$ such that $\chi_j = zq^{a_j}$ for some $a_j \in \mathbf{Z}$, for $j = 1, \dots, m$, and any m. The set L_z will be called a line.

(ii) Let Δ_1 and Δ_2 be segments in the same line L_z, say

$$\Delta_1 = [q^{a+l-1}z, \dots, q^a z], \qquad \Delta_2 = [q^{b+m-1}z, \dots, q^b z].$$

We will say that Δ_1 *precedes* Δ_2 if either $a + l - 1 < b + m - 1$ or if $a + l - 1 = b + m - 1$ and $a \leqq b$. We will say that Δ_1 and Δ_2 are *linked* if one of the following conditions is satisfied:

$$a + l - 1 \geqq b + m \geqq a > b \quad \text{or} \quad b + m - 1 \geqq a + l \geqq b > a.$$

If Δ_1 and Δ_2 are linked and Δ_1 precedes Δ_2, set $\Delta_1 \cap \Delta_2 = [q^{b+m-1}, \dots, q^a]$ and $\Delta_1 \cup \Delta_2 = [q^{a+l-1}, \dots, q^b z]$, and if Δ_1 and Δ_2 are linked but Δ_2 precedes Δ_1, set $\Delta_1 \cup \Delta_2 = \Delta_2 \cap \Delta_1$ and $\Delta_1 \cup \Delta_2 = \Delta_2 \cup \Delta_1$.

(iii) We put a partial order on L_z as follows. Let $\chi = [q^{a_1}z, \dots, q^{a_l}z]$, $\chi' = [q^{b_1}z, \dots, q^{b_m}z] \in L_z$ and define $\chi > \chi'$ if $l = m$ and the sequence $(a_1, \dots, a_l)$ is lexicographically bigger than $(b_1, \dots, b_k)$, i.e., for some k, $a_j = b_j$ for $j < k$ and $a_k > b_k$.

(iv) A character $\chi \in \mathcal{C}$ will be called *reduced* if it is of the form $(\psi_1, \dots, \psi_t)$ where the ψ_j are sequences belonging to distinct lines. Call $\chi = (\psi_1, \dots, \psi_t)$ the decomposition of χ into lines.

For $\chi \in \mathscr{C}$, let $\mathscr{I}(\chi)$ be the set of irreducible constituents of $I(\chi)$. As shown in [4], for $\mathscr{I}(\chi_1)=\mathscr{I}(\chi_2)$ if $\chi_1,\chi_2 \in \mathscr{C}$ and $\chi_1 \in W_{\chi_2}$, while $\mathscr{I}(\chi_1)\cap\mathscr{I}(\chi_2)=\varnothing$ otherwise. Since every irreducible $\mathscr{H}$-module is a constituent of $\mathscr{I}(\chi)$ for some $\chi \in \mathscr{C}$, it will suffice to describe the sets $\mathscr{I}(\chi)$ for χ a representative of a given W-orbit in $\mathscr{C}$.

Therefore, *we fix a character* $\eta \in \mathscr{C}$ *for the rest of the paper.* We assume that η is reduced with line decomposition $\eta=(\eta^1,\dots,\eta^t)$. Suppose that η^j is a sequence of length m_j, so that $(m_1,\dots,m_t)$ is a partition of n. Let T be the subset of S associated to this partition and set $\mathscr{O}(\eta)=\{w_\eta : w \in W_T\}$. We put a total order on $\mathscr{O}(\eta)$ as follows. If $\chi_j=(\chi_j^1,\dots,\chi_j^t)$, for $j=1,2$, are line decompositions of elements of $\mathscr{O}(\eta)$, then $\chi_1>\chi_2$ if for some k, $\chi_1^i=\chi_2^i$ for $i<k$ and $\chi_1^k>\chi_2^k$.

If $\chi \in \mathscr{C}$, there is a unique sequence of segments $\Delta_1,\dots,\Delta_r$ such that $\chi=(\Delta_1,\dots,\Delta_r)$ and r is as small as possible. Call $(\Delta_1,\dots,\Delta_r)$ the decomposition of χ into segments.

Definition 3.2. A character $\chi \in \mathscr{O}(\eta)$ with segment decomposition $(\Delta_1,\dots,\Delta_r)$ is called *min-reduced* if Δ_i precedes Δ_{i+1} for all i such that Δ_i and Δ_{i+1} lie on the same line. Let $M\mathscr{O}(\eta)$ denote the set of min-reduced elements in $\mathscr{O}(\eta)$.

The set $M\mathscr{O}(\eta)$ inherits a total order from $\mathscr{O}(\eta)$. We also define a partial order on the set of collections of segments $\Phi=\{\Delta_1,\dots,\Delta_t\}$. Let $\lesssim$ be the partial order generated by the relations $\Phi'\lesssim\Phi$ where Φ' is obtained from Φ by replacing two linked segments $\Delta_i,\Delta_j \in \Phi$ by $\Delta_i \cap \Delta_j$ and $\Delta_i \cup \Delta_j$.

The remaining sections of the paper will be devoted to proving the following theorems.

Theorem 3.3. *Let* $\chi \in M\mathscr{O}(\eta)$ *have segment decomposition* $\chi=(\Delta_1,\dots,\Delta_r)$ *and let* $\Phi=\{\Delta_1,\dots,\Delta_r\}$. *Then* $I(\Phi)$ *has a unique irreducible quotient* M *and* $\chi \leqq \psi$ *for all* $\psi \in P(M)\cap\mathscr{O}(\eta)$ *under the total order* $\leqq$ *on* $\mathscr{O}(\eta)$.

Theorem 3.4. (1) *Let* $M \in \mathscr{I}(\eta)$. *Then* $P(M)\cap\mathscr{O}(\eta)\neq\varnothing$ *and the unique minimal element* $\chi_M \in P(M)\cap\mathscr{O}(\eta)$ *(for the order* $\leqq$*) lies in* $M\mathscr{O}(\eta)$.

(2) *The map*:

$$\mathscr{I}(\eta)\to M\mathscr{O}(\eta)$$

$$M \to \chi_M$$

is a bijection.

Theorem 3.5. *Let* $\chi,\psi' \in M\mathscr{O}(\eta)$. *Let* $\chi=(\Delta_1,\dots,\Delta_r)$ *be the segment decomposition of* χ *and set* $\Phi=\{\Delta_1,\dots,\Delta_r\}$. *Let* M *be the irreducible* $\mathscr{H}$*-module* *corresponding to ψ' by Theorem* 3.4. *Let $\psi' = (\Delta'_1, \ldots, \Delta'_s)$ be the segment decomposition of ψ', and set $\Phi' = \{\Delta'_1, \ldots, \Delta'_s\}$. Then M is a constituent of $I(\Phi)$ if and only if $\Phi' \lesssim \Phi$.*

§4. In this section we show that the proofs of Theorems 3.3, 3.4, and 3.5 can be reduced to the case that η lies in a line.

Let $\mathcal{A}^W$ be the subalgebra of W-invariants in $\mathcal{A}$. By a theorem of Bernstein (see [3] for a proof), $\mathcal{A}^W$ is the center of W. The characters of $\mathcal{A}^W$ correspond to W-orbits in $\mathcal{A}$. For $[\chi] = \{w_\chi : w \in W\}$ a W-orbit in $\mathcal{C}$ and $M \in \mathcal{M}$, set

$$M[\chi] = \{m \in M : (a - \chi(a))^t m = 0 \text{ for all } a \in \mathcal{A}^W, \text{some } t \in \mathbf{Z}^+\}.$$

It is clear that $M[\chi]$ is $\mathcal{H}$-stable and that

$$M = \bigoplus_{\chi \in \mathcal{C}/W} M[\chi].$$

If T is a subset of S, then it follows that the center of $\mathcal{H}_T$ is the subalgebra $\mathcal{A}^{W_T}$ of W_T-invariants in $\mathcal{A}$. Hence for all $\chi \in \mathcal{C}/W_T$, the space

$$M[\chi, T] = \{m \in M : (a - \chi(a))^t m = 0 \text{ for all } a \in \mathcal{A}^{W_T}, \text{some } t \in \mathbf{Z}^+\}$$

is $\mathcal{H}_T$-stable and

$$M = \bigoplus_{\chi \in \mathcal{C}/W_T} M[\chi, T].$$

Let $M \in \mathcal{M}$ satisfy $P(M) \subseteq W\eta$. Set

$$M_{\mathrm{red}} = \bigoplus_{\chi \in \mathcal{O}(\eta)} M_\chi^{\mathrm{gen}}.$$

By the results of the previous paragraph, M_{red} is an $\mathcal{H}_T$ submodule of M, where T is the subset associated with the partition $(m_1, \ldots, m_t)$ defined by η (see §3). This follows because $\mathcal{O}(\eta) = W_T\eta$. Thus we have a map of $\mathcal{H}$-modules:

$$f : \mathcal{H} \bigoplus_{\mathcal{H}_T} M_{\mathrm{red}} \to M,$$

$$T \otimes m \to Tm.$$

Proposition 4.1. *The map f is an isomorphism.*

Proof. The functor $M \to M_{\mathrm{red}}$ from $\mathcal{H}$-modules to $\mathcal{H}_T$-modules is exact. The functor $N \to \mathcal{H} \otimes_{\mathcal{H}_T} N$ from $\mathcal{H}_T$-modules to $\mathcal{H}$-modules is also exact and we have:

$$\dim(\mathcal{H} \bigotimes_{\mathcal{H}_T} N) = (\dim(N))|W/W_T|,$$

since $\mathcal{H} \otimes_{\mathcal{H}_T} N \xrightarrow{\sim} \mathcal{H}_W \otimes_{\mathcal{H}_{W_T}} N$ as vector spaces. It therefore suffices to prove the proposition for M irreducible. If M is irreducible, it will follow that f is an isomorphism if we prove that

$$(*) \qquad \dim(M) = \dim(M_{\mathrm{red}}) | W / W_T |.$$

Let $\{w_1, \ldots, w_l\}$ be the set of representatives for W/W_T such that for all $j = 1, \ldots, l$, $l(w_j z) \leqq l(w_j)$ for all $z \in W_T$. Then $W\eta = \{w_j \phi : \phi \in \mathcal{O}(\eta),\ j = 1, \ldots, l\}$. To prove $(*)$, it will suffice to show that $\dim(M^{\mathrm{gen}}_\phi) = \dim(M^{\mathrm{gen}}_{w_j\phi})$ for all $j = 1, \ldots, l$. This follows easily from the next lemma.

LEMMA 4.2. *Let* $M \in \mathcal{M}$, $\chi \in \mathcal{C}$, *and suppose that* $\chi_j \neq q^{\pm 1} \chi_{j+1}$. *Then* $\dim(M^{\mathrm{gen}}_\chi) = \dim(M^{\mathrm{gen}}_{s_j\chi})$.

PROOF. Let $\mathcal{H}_2$ be the subalgebra of $\mathcal{H}$ generated by T_{s_j}, $x_j^{\pm 1}$, and $x_{j+1}^{\pm 1}$. The subspace $M^{\mathrm{gen}}_\chi \oplus M^{\mathrm{gen}}_{s_j\chi}$ is stable under $\mathcal{H}_2$ and, as an $\mathcal{H}_2$-module, all of its constituents are constituents of the $\mathcal{H}_2$-module $I(\chi')$, where $\chi' = [\chi_j, \chi_{j+1}]$. If $\chi_j \neq q^{\pm 1} \chi_{j+1}$, then $I(\chi')$ is irreducible ([4], Corollary 3.2) and the lemma follows.

Proposition 4.1 shows that for the proofs of the theorems of §3, it is sufficient to look at the case where η lies in a line.

§5. Analysis of the product of two segments

We begin with a lemma concerning the case $n = 3$.

LEMMA 5.1. *Let* $n = 3$ *and let* $\chi = [1, 1, q] \in \mathcal{C}$. *Then* $\mathcal{I}(\chi)$ *consists of two irreducible modules of dimension three. The weight* $[1, q, 1]$ *occurs with multiplicity one in each of them. One of them contains* χ *with multiplicity two and the other contains* $[q, 1, 1]$ *with multiplicity two.*

PROOF. Let $\Phi = \{[1], [q, 1]\}$. Then $I(\Phi)$ is a three-dimensional submodule of $I(\chi)$, contains the weight $[1, q, 1]$ with multiplicity one, and the weight $[q, 1, 1]$ with multiplicity two. The lemma follows immediately if we show that $I(\chi)$ contains no one-dimensional constituents. However, using the relations defining $\mathcal{H}$, it is easy to show that (for any n), if $\tau : \mathcal{H} \to \mathbf{C}$ is a character, then the restriction of τ to $\mathcal{A}$ is of the form (Δ) or $(\tilde{\Delta})$ for Δ a segment of length n.

For the rest of this section, let $\Delta_1 = [q^{a+l-1}, \ldots, q^a]$ and $\Delta_2 = [q^{b+m-1}, \ldots, q^b]$ be segments of length l and m, respectively, such that $l + m = n$. Let $T = \{s_j : j \neq l\}$ be the subset of S associated to the partition (l, m) of n. Let $M = \Delta_1 \times \Delta_2$ and $N = \Delta_2 \times \Delta_1$, and set $\chi = (\Delta_1, \Delta_2)$, $\chi' = (\Delta_2, \Delta_1)$.

PROPOSITION 5.2. *If Δ_1 and Δ_2 are not linked, then M is irreducible and is isomorphic to N.*

PROOF. By the results of §6 of [4], there is a non-zero map from M to N. Hence it will suffice to prove the irreducibility of M or N.

We may assume that $\Delta_1 \subseteq \Delta_2$ or $\Delta_2 \subseteq \Delta_1$, for if this is not the case and if Δ_1 and Δ_2 are not linked, then the weight spaces of M are all one-dimensional (see proposition 4.5 of [4]) and the operators $A_w(\chi)$ are invertible for all $w \in W(T)$. The irreducibility of M then follows immediately.

In §3 of [4], a character ψ was defined to be special if its stabilizer in W is of the form $W_{T'}$ for some subset $T' \subseteq S$. By theorem 3.1 of [4], if ψ is special, then $\dim R_\psi \leqq 1$ for any submodule R of $I(\psi')$ for any $\psi' \in W\psi$. It is easy to see that $P(M)$ contains a unique special weight $\psi_0 \in W(T)\chi$. The irreducibility of M is thus a consequence of the following two facts:

(a) If L is a non-zero submodule of M, then $L_{\psi_0} \neq 0$, and hence $L_{\psi_0} = M_{\psi_0}$.

(b) M is generated by M_{ψ_0}.

We first prove (b). Set ${}^-\Delta_1 = [q^{a+l-2}, \ldots, q^a]$ and $\Delta_2^- = [q^{b+m-1}, \ldots, q^{m+1}]$. Since we are free to interchange Δ_1 and Δ_2 (it suffices to prove the irreducibility of M or N), we may assume that either ${}^-\Delta_1 \supseteq \Delta_2$, or $\Delta_1 = \Delta_2$, or $\Delta_1 \subseteq \Delta_2^-$. In the first two cases, let $\mathcal{H}_{n-1}$ be the subalgebra of $\mathcal{H}$ generated by the T_{s_j} with $j > 1$ and $x_2^{\pm 1}, \ldots, x_n^{\pm 1}$. Under the action of $\mathcal{H}_{n-1}$, the canonical generator C_{w_T} of M generates an $\mathcal{H}_{n-1}$-module isomorphic to ${}^-\Delta_1 \times \Delta_2$ and this subspace of M contains M_{ψ_0}. By induction on n, $C_{w_T} \in \mathcal{H}_{n-1} M_{\psi_0}$. The case $\Delta_2^- \supseteq \Delta_1$ is similar.

For the proof of (a), we use the following lemma.

LEMMA 5.3. *Let $\chi \in \mathcal{C}$ and let M be a submodule of $I(\chi)$. Let $w \in W$ and assume that M_{w_χ} contains an element of the form*

$$m = C_w + \sum_{y \not\geq w} \alpha_y C_y \qquad (\alpha_y \in \mathbf{C}).$$

Then for all $s \in S$ such that $sw > w$ and $sw_\chi \neq \chi$, $M_{sw_\chi} \neq 0$ and contains an element of the form:

$$m' = C_{sw} + \sum_{y \not\geq sw} \alpha'_y C_y \qquad (\alpha_y \in \mathbf{C}).$$

PROOF. If $sw_\chi \neq w_\chi$, then $A_s(w_\chi)$ is defined and it will suffice to show that $A_s(w_\chi)m \neq 0$. We have $A_s(w_\chi) = m_s(w_\chi) + C_s$ and hence

$$A_s(w_\chi)m = C_sC_w + \sum_{y \not\geq w} \alpha_y C_s C_y + m_s(w_\chi)m.$$

From the multiplication rules for C_sC_w (see [1] or [4]), it follows that $A_s(w_\chi)m$ is of the form

$$C_{sw} + \sum_{y \not\geq sw} \alpha'_y C_y$$

if $sw > w$.

For the proof of (a), we assume that $\Delta_1 \supseteq \Delta_2$; the case $\Delta_1 \subseteq \Delta_2$ is similar.

To prove (a), we use the following notation. Write $\Delta_1 = [q^{a+l-1}, \ldots, q^a]$ and $\Delta_2 = [\underline{q}^{b+m-1}, \ldots, \underline{q}^b]$. A character $\psi = w_\chi$ for some $w \in W(T)$ will be written typically as

$$\psi = [q^{i_1}, \underline{q}^{i_2}, \underline{q}^{i_3}, \ldots, q^{i_n}].$$

Here the location of the bars below the powers of q determine uniquely the element $w \in W(T)$ such that $\psi = w_\chi$ because $W(T)$ consists of permutations such that w^{-1} preserves the order between the elements within one of the Δ_j (note that in general, $w_\chi = [\chi_{w^{-1}(1)}, \ldots, \chi_{w^{-1}(n)}]$ if $\chi = [\chi_1, \ldots, \chi_n]$).

Now let L be a non-zero submodule of M. We will say that w_χ occurs in L for $w \in W(T)$ if L_{w_χ} contains an element of the form

$$C_w + \sum_{y \not\geq w} \alpha_y C_y \qquad (\alpha_y \in \mathbf{C}).$$

Our strategy is to start with any w_χ occurring in L and use Lemma 5.3 and the operators $A_s(w_\chi)$ to obtain $L_{\psi_0} \neq 0$. So assume that w_χ occurs in L. If $\chi_{w^{-1}(j)} = q^\gamma w_{w^{-1}(j+1)}$ with $\gamma < -1$, then $A_{s_j}(w_\chi)$ is invertible and y_χ occurs in L for some y such that $y_\chi = s_j w_\chi$. Hence we may assume that $\chi_{w^{-1}(j)} = q^\gamma \chi_{w^{-1}(j+1)}$ with $\gamma \geqq -1$ for all j. We are concerned only with those j such that $\gamma = -1$, for if none exist, then $w_\chi = \psi_0$ and we are done. Set $d_j = \chi_{w^{-1}(j)}$. If $(d_j, d_{j+1}) = (q^{r-1}, \underline{q}^r)$, then $s_j w > w$ and Lemma 5.3 implies that $s_j w$ occurs in L. So we may assume that those j with $\gamma = -1$ satisfy $(d_j, d_{j+1}) = (\underline{q}^{r-1}, q^r)$. Here we use the fact that $\underline{q}^l$ (resp. q^l) cannot precede $\underline{q}^k$ (resp. q^k) if $l < k$ because all $w \in W(T)$ preserve order in the blocks Δ_1, Δ_2.

Consider the largest j such that $\gamma = -1$. For this j we have $(d_j, d_{j+1}, d_{j+2}) = (\underline{q}^{r-1}, q^r, q^{r-1})$ since $\Delta_1 \supseteq \Delta_2$. Note that $s_{j+1} w_\chi$ is not a weight of M since no subsequence of the form (q^{r-1}, q^{r-1}, q^r) occurs in any weight of M. Therefore Lemma 5.1 implies that $s_j w_\chi$ is a weight of L; this is seen by considering the $\mathcal{H}_3$-submodule generated by $L_{s_j w_\chi}$, where $\mathcal{H}_3$ is the subalgebra of $\mathcal{H}$ generated by T_{s_j}, $T_{s_{j+1}}$ and $x_j^{\pm 1}$, $x_{j+1}^{\pm 1}$, $x_{j+3}^{\pm 1}$. Now $s_j w_\chi$ is "closer" to ψ_0 than w_χ. Continuing in this way we obtain that $L_{\psi_0} \neq 0$.

PROPOSITION 5.4. *Assume that Δ_1 and Δ_2 are linked. Then M and N are indecomposable. Up to multiples, there is a unique non-zero map $\phi : M \to N$.*

(i) *If Δ_1 precedes Δ_2, then* $\mathrm{Ker}(\phi)$ *is isomorphic to* $(\Delta_1 \cap \Delta_2) \times (\Delta_1 \cup \Delta_2)$ *and* $M/\ker(\phi)$ *is an irreducible $\mathcal{H}$-module which we will denote by* $\langle \Delta_1, \Delta_2 \rangle$.

(ii) *If Δ_2 precedes Δ_1, then* $\mathrm{Ker}(\phi)$ *is isomorphic to* $\langle \Delta_2, \Delta_1 \rangle$ *and* $M/\ker(\phi)$ *is isomorphic to* $(\Delta_1 \cap \Delta_2) \times (\Delta_1 \cup \Delta_2)$.

PROOF. Assume first that Δ_1 precedes Δ_2. There is a unique $w \in W(T)$ such that $w_\chi = (\Delta_1 \cap \Delta_2, \Delta_1 \cup \Delta_2)$. In particular, $\dim(M_{w_\chi}) = 1$. By Lemma 4.6 of [4], there is a unique element $m \in M_{w_\chi}$ of the form:

$$m = C_{ww_T} + \sum_{\substack{z < ww_T \\ zw_T^{-1} \in W(T)}} \alpha_z C_z.$$

For $j \neq |\Delta_1 \cap \Delta_2|$, $m_{s_j}(s_j w_\chi) = 0$ and hence $A_{s_j}(w_\chi) = C'_{s_j}$. For $j \neq |\Delta_1 \cap \Delta_2|$, $|\Delta_1 \cap \Delta_2| + |\Delta_2|$, $s_j w_\chi$ is not a weight of M and hence $C'_{s_j} m = 0$. We will show below that $C'_{s_j} m = 0$ for $j = |\Delta_1 \cap \Delta_2| + |\Delta_2|$. Assuming this, we obtain, by proposition 4.5 of [4], a non-zero map from $(\Delta_1 \cap \Delta_2) \times (\Delta_1 \cup \Delta_2)$ to M which sends its canonical generator to m. Since $(\Delta_1 \cap \Delta_2) \times (\Delta_1 \cup \Delta_2)$ is irreducible by Proposition 5.2, m generates a submodule of M which is isomorphic to $(\Delta_1 \cap \Delta_2) \times (\Delta_1 \cup \Delta_2)$. Let M' be the quotient $M/\mathcal{H}m$. It can be checked that all weights of M' occur with multiplicity one and that for all $\psi \in P(M')$ and $s \in S$ such that $s\psi \in P(M)$, $A_s(\psi)$ is invertible. It follows that M' is irreducible. In addition, M is indecomposable because M is generated by an element of weight χ (its canonical generator), but the submodule M'' of M isomorphic to $(\Delta_1 \cap \Delta_2) \times (\Delta_1 \cup \Delta_2)$ contains no elements of weight χ. By the results of §6 of [4] (specifically, proposition 6.4 of [4], whose proof does not rely on the results of this paper), there are non-zero maps $\phi : M \to N$ and $\phi' : N \to M$. Since N is generated by an element of weight $\chi' = (\Delta_2, \Delta_1)$ and $M_{\chi'} \subseteq M''$, M and N are not isomorphic. Parts (i) and (ii) follow. Finally, N is indecomposable because $N_{\chi'}$ generates N by χ' does not occur as a weight of the submodule of N isomorphic to M'.

It remains to verify that, in the above notation $C'_{s_j} m = 0$ for $j = |\Delta_1 \cap \Delta_2| + |\Delta_2|$. Note that $s_j w < w$ for this j. By lemma 6.6 of [4],

$$A_{s_j}(w_\chi) m = C'_{s_j} m = \sum \alpha'_z C_z$$

for some $\alpha'_z \in \mathbf{C}$, where the sum is over $z \neq s_j w w_T$, ww_T such that $z < ww_T$ or $sz < ww_T$ and $zw_T^{-1} \in W(T)$. For such z, $zw_T^{-1}\chi \neq s_j w_\chi$ and hence $A_{s_j}(w_\chi)m$ cannot have weight $s_j w_\chi$ (this is obvious, for example, from the proof of lemma 4.6 of [4]). Therefore $A_{s_j}(w_\chi)m = C'_{s_j} m = 0$.

§6. In this section, we complete the proofs of Theorems 3.3, 3.4 and 3.5. By the results of §4, we may assume that η lies on a line and there is no loss of generality in assuming that $\eta \in L_q$. For η on a line, $\mathcal{O}(\eta) = W\eta$ and thus the first assertion of part (i) of Theorem 3.4 is clear. For $M \in \mathcal{I}(\eta)$, let χ_M be the unique minimal element of $P(M)$.

PROPOSITION 6.1. *For $M \in \mathcal{I}(\eta)$ and let χ_M have a segment decomposition $\chi_M = (\Delta_1,\ldots,\Delta_t)$. Then M is a quotient of $\Delta_1 \times \cdots \times \Delta_t$.*

PROOF. Let $|\Delta_i| = n_i$ and let T be the subset of S associated to the partition $(n_1,\ldots,n_t)$ of n. By definition of χ_M, $s_j\chi_M \not\in P(M)$ for all $s_j \in T$. Since $m_{s_j}(s_j\chi_M) = 0$ for all $s_j \in T$, $A_{s_j}(\chi_M) = C'_{s_j}$ and thus $C'_{s_j}M_{\chi_M} = 0$ for all $s_j \in T$. The proposition follows from proposition 4.5 of [4].

PROPOSITION 6.2. *Let $M \in \mathcal{I}(\eta)$. Then χ_M is min-reduced.*

PROOF. Let χ_M have a segment decomposition $\chi_M = (\Delta_1,\ldots,\Delta_t)$, so that M is a quotient of $\Delta_1 \times \cdots \times \Delta_t$ by Proposition 6.1. If χ_M is not min-reduced, then Δ_{j+1} precedes Δ_j for some j. If Δ_j and Δ_{j+1} are not linked, then $\Delta_j \times \Delta_{j+1}$ is isomorphic to $\Delta_{j+1} \times \Delta_j$ by Proposition 5.2. Hence M is also a quotient of $\Delta_1 \times \cdots \times \Delta_{j+1} \times \Delta_j \times \cdots \times \Delta_t$ and $(\Delta_1,\ldots,\Delta_{j+1},\Delta_j,\ldots,\Delta_t)$ occurs as a weight of M. It is smaller than χ_M, contradicting the minimality of χ_M. If Δ_j and Δ_{j+1} are linked, then Proposition 5.4 shows that M is also a quotient of $\Delta_1 \times \cdots \times (\Delta_j \cap \Delta_{j+1}) \times (\Delta_j \cup \Delta_{j+1}) \times \cdots \times \Delta_t$ and again, $(\Delta_1,\ldots,\Delta_j \cap \Delta_{j+1},\Delta_j \cup \Delta_{j+1},\ldots,\Delta_t)$ is smaller than χ_M and occurs as a weight of M.

LEMMA 6.3. *Let $\Delta = [q^{m-1},\ldots,1]$ be a segment and let $\Delta_j = [q^j,\ldots,1]$ for $j = 0,\ldots,m-1$. Let*

$$M = \Delta_j \times \underbrace{\Delta \times \cdots \times \Delta}_{t\text{-times}}.$$

Then M is irreducible.

PROOF. Let $\psi = (\Delta_j,\Delta,\ldots,\Delta)$. Then ψ is the unique min-reduced weight in $P(M)$. Thus, if N is a non-zero irreducible submodule of M, then Propositions 6.1 and 6.2 imply that N is a quotient of M. The lemma will follow if we show that every non-zero element of M_ψ generates M. Let ψ' be the unique special weight in $P(M)$. By theorem 3.1 of [4], $\dim M_{\psi'} = 1$. If we show that $M_{\psi'}$ generates M, it will follow that $N_{\psi'} \neq 0$, hence $N_{\psi'} = M_{\psi'}$ and again $N = M$, since N is a quotient of M. Using these two ways of establishing the lemma, we show that it follows by induction on $n = j + 1 + tm$. So assume the lemma holds for

$n-1$ and let $\mathcal{H}_{n-1}$ be the subalgebra of $\mathcal{H}$ generated by $T_{s_2},\ldots,T_{s_n}$ and $x_2^{\pm 1},\ldots,x_n^{\pm 1}$. Let C be the canonical generator of M.

If $0 \leqq j < m-1$, then $\mathcal{H}_{n-1}C$ is an $\mathcal{H}_{n-1}$-submodule of M isomorphic to $\Delta_{j-1}\times\Delta\times\cdots\times\Delta$ (let $\Delta_{j-1}=\varnothing$ if $j=0$) and it contains M_ψ. By induction, each non-zero element of M_ψ generates $\mathcal{H}_{n-1}C$ under the action of $\mathcal{H}_{n-1}$ and hence generates M under $\mathcal{H}$. If $j=m-1$, then $\mathcal{H}_{n-1}C$ contains an element of weight ψ', hence $\mathcal{H}_{n-1}C$ contains $M_{\psi'}$. Again by induction, $\mathcal{H}_{n-1}M_{\psi'}=\mathcal{H}_{n-1}C$ and so $M_{\psi'}$ generates M under $\mathcal{H}$.

PROPOSITION 6.4. *Let $\chi\in M\mathcal{O}(\eta)$ have a segment decomposition $\chi=(\Delta_1,\ldots,\Delta_t)$. Then $\Delta_1\times\cdots\times\Delta_t$ has a unique irreducible quotient M and $\chi_M=\chi$.*

PROOF. This first statement will follow if we show that every non-zero element in the χ-weight space of $\Delta_1\times\cdots\times\Delta_t$ generates $\Delta_1\times\cdots\times\Delta_t$, for then $\Delta_1\times\cdots\times\Delta_t$ has a unique maximal submodule (the submodule N which is maximal subject to the condition $N_\chi=0$). Let $|\Delta_j|=n_j$ and let T be the subset of S associated to the partition $(n_1,\ldots,n_t)$ of n. If $z\in W(T)$ and $z_\chi=\chi$, then z can only act by changing equal segments Δ_j amongst themselves. Let $\mathcal{H}'$ be the subalgebra of $\mathcal{H}$ generated by $\mathcal{A}$ and the T_{s_j} for all j except those of the form $j=\sum_{i=1}^{l} n_i$ for those l such that $\Delta_l\neq\Delta_{l+1}$. Then the χ-weight space of $\Delta_1\times\cdots\times\Delta_t$ is contained in the $\mathcal{H}'$-submodule $\mathcal{H}'C$, where C is the canonical generator of $\Delta_1\times\cdots\times\Delta_t$. The algebra $\mathcal{H}'$ is isomorphic to $\mathcal{H}_{m_1}\times\cdots\times\mathcal{H}_{m_r}$ for some partition $(m_1,\cdots,m_r)$ of n, where $\mathcal{H}_{m_i}$ is the Hecke algebra for $\mathrm{GL}_{m_i}(F)$. The $\mathcal{H}'$-module $\mathcal{H}'C$ is isomorphic to the tensor product of $\mathcal{H}_{m_i}$-modules of the form $\Delta\times\Delta\times\cdots\times\Delta$. By Lemma 6.3, $\mathcal{H}'C$ is therefore an irreducible $\mathcal{H}'$-module. This proves the first statement and the second follows because χ is the minimal element of $P(\Delta_1\times\cdots\times\Delta_t)$.

Propositions 6.1, 6.2, and 6.4 complete the proofs of Theorems 3.3 and 3.4. It remains to prove Theorem 3.5.

For any $\mathcal{H}$-module M, define the formal character

$$\mathrm{ch}(M)=\sum_{\chi\in\mathscr{C}}(\dim M_\chi^{\mathrm{gen}})\chi$$

as an element of the integral group ring $\mathbf{Z}[\mathscr{C}]$, as in [4]. From Theorem 3.4, it follows that the set of irreducible factors in a composition series for M is uniquely determined by $\mathrm{ch}(M)$; one uses the fact that an irreducible $\mathcal{H}$-module N is uniquely determined by its minimal weight χ_N and the partial order on the set of such weights.

From now on, we use the notation of the statement of Theorem 3.5. According to theorem 6.5 of [4] (whose proof is independent of Theorem 3.5), there is a filtration $\{I(\Phi)^k\}$ of $I(\Phi)$ such that

$$\sum_{k>0} \mathrm{ch}(I(\Phi)^k) = \sum_{\substack{i<j \\ \Delta_i,\Delta_j \text{ linked}}} \mathrm{ch}(I(\Phi(i,j))).$$

Here $\Phi(i,j)$ denotes the collection of segments (in any order) obtained by replacing a linked pair of segments Δ_i and Δ_j in Φ by $\Delta_i \cap \Delta_j$ and $\Delta_i \cup \Delta_j$. It follows that M is a constituent of $I(\Phi)$ whenever $\Phi' \lesssim \Phi$ by induction.

The only if part of Theorem 3.5 follows from Proposition 6.2 and the following purely combinatorial assertion: if a min-reduced character ψ' is a weight of $I(\Phi)$, then $\Phi' \lesssim \Phi$ (in the notation of Theorem 3.5).

Let ψ'' be a character with segment decomposition $\psi'' = (\Delta''_1, \ldots, \Delta''_t)$. Suppose that $\Delta''_i = [q^{a+l-1}, \ldots, q^a]$ and $\Delta''_{i+1} = [q^{b+k-1}, \ldots, q^b]$. If $a + l - 1 = b + k - 1$, we will say that Δ''_i and Δ''_{i+1} have the same starting point. If the condition $a + l - 1 \leqq b + k - 1$ is satisfied for all $i = 1, \ldots, t-1$, we will say that ψ'' is semi-reduced.

Now weaken the assumption on ψ' and suppose only that ψ' is semi-reduced. We will show that $\Phi' \lesssim \Phi$ if ψ' occurs as a weight of $I(\Phi)$. Let $\chi = [\chi_1, \ldots, \chi_n]$ and $\psi' = [\chi_{i_1}, \ldots, \chi_{i_n}]$. Then ψ' is obtained from χ by permuting the χ_j so that the order among entries of a segment Δ_k is preserved. First consider the case that $\Delta_1 = \Delta'_1$. Then by induction on n, $\{\Delta'_2, \ldots, \Delta'_s\} \lesssim \{\Delta_2, \ldots, \Delta_r\}$ and hence $\Phi' \lesssim \Phi$.

Now let $\Delta_1 = [\chi_1, \ldots, \chi_a]$ and ψ'' be the character obtained from ψ' by moving the entries $\chi_1, \ldots, \chi_a$ occurring among the χ_{i_k} to the extreme left but preserving the order among the other entries. Thus $\psi'' = [\chi_1, \ldots, \chi_a, \chi_{j_1}, \ldots, \chi_{j_{n-a}}]$ and ψ'' is a also a semi-reduced weight of $I(\Phi)$. Let $\psi'' = (\Delta''_1, \ldots, \Delta''_p)$ be the segment decomposition of ψ'' and let $\Phi'' = \{\Delta''_1, \ldots, \Delta''_p\}$. Thus $\Delta''_1 = \Delta_1$. By the case considered in the previous paragraph, $\Phi'' \lesssim \Phi$. It will therefore suffice to show that $\Phi' \lesssim \Phi''$. We have that ψ' is obtained from ψ'' by a permutation which preserves the order among the entries of $(\Delta''_2, \ldots, \Delta''_p)$. It is easy to see that Δ''_1 can be decomposed into smaller segments, $\Delta''_1 = (\Delta^1, \ldots, \Delta^k)$ so that ψ' is obtained from ψ'' by inserting the Δ^i consecutively among the Δ''_j. Except possibly for Δ^i, if (Δ''_j, Δ^i) occurs in ψ', then (Δ''_j, Δ^i) is itself a segment. It follows easily that $\Phi' \lesssim \Phi''$.

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